6.6/53 $F(x) = \int_2^x \sqrt{3t^2+1}\,dt$

a) $F(2) = \int_2^2 \sqrt{3t^2+1}\,dt = 0$

b) $F'(2) = \underbrace{\frac{d}{dx}\left(\int_2^x \sqrt{3t^2+1}\,dt\right)}_{\text{FTC \#1}}\Big|_{x=2}$

$= \sqrt{3x^2+1}\,\Big|_{x=2} = \sqrt{13}$

$F'(2)$ find $F'(x)$ then replace x w/ 2

c) $F''(2) = \frac{d}{dx}\left(F'(x)\right)\Big|_{x=2} = \underbrace{\frac{d}{dx}\left(\sqrt{3x^2+1}\right)}_{\text{chain rule}}\Big|_{x=2}$

$= \frac{1}{2}(3x^2+1)^{-\frac{1}{2}}(6x)\Big|_{x=2} = \frac{6 \cdot 2}{2\sqrt{13}} = \frac{6}{\sqrt{13}}$

Jan 9-7:27 AM

6.6/55) $F(x)=\int_0^x \frac{t-3}{t^2+7}\,dt$ over $(-\infty,\infty)$

a) find where $F(x)$ has a minimum.

$F'(x)=\frac{d}{dx}\left(\int_0^x \frac{t-3}{t^2+7}\,dt\right)=\frac{x-3}{x^2+7}$

F' und : no xs

$F'=0$ $\frac{x-3}{x^2+7}=0$ so $x-3=0$

or $x=+3$

critical pt

sign chart of $F'(x)$

- - - - | + + + + +

0 3 5

∴ $F(x)$ has a relative minimum at $x=3$.

its also an absolute minimum.

b) $F(x)$ is decreasing on $(-\infty,3]$

$F(x)$ is increasing on $[3,\infty)$

c) $F''(x)=\frac{d}{dx}(F'(x))=\frac{d}{dx}\left(\frac{x-3}{x^2+7}\right)=\frac{(1)(x^2+7)-(x-3)(2x)}{(x^2+7)^2}$

$=\frac{x^2+7-2x^2+6x}{(x^2+7)^2}=-\frac{x^2-6x-7}{(x^2+7)^2}$

F'' never und.

$F''=0 \Rightarrow x^2-6x-7=0$

$(x+1)(x-7)=0$

$x-7=0 \Rightarrow x=+7$

$x+1=0 \Rightarrow x=-1$

Sign chart of $F''(x)$ $F''(x)=-\frac{(x+1)(x-7)}{(x^2+7)^2}$

- - - | + + + | - - -

-5 -1 0 +7 10

c-up: $(-1,7)$

c-down: $(-\infty,-1)\cup(7,\infty)$

6.7) Rectilinear Motion, part 2

Recall if $s(t)$ describes the (1-dimensional) position of an object, $s(t) = \int v(t)\,dt$

then $v(t) = s'(t)$ $v(t) = \int a(t)\,dt$

is the velocity of the object,

and $a(t) = v'(t) = s''(t)$ is the acceleration of the object.

Jan 9-7:28 AM

Let's assume we know $a(t) = -32$.

$g = -32$ fps $= -9.8$ m/s

$$V(t) = \int a(t)\,dt = \int -32\,dt$$

$$= -32t + C$$

a velocity [starting or initial]

$$= -32t + V_0$$

$$s(t) = \int v(t)\,dt = \int -32t + V_0\,dt$$

$$= -32\frac{t^2}{2} + V_0 t + C$$

$$= -32\frac{t^2}{2} + V_0 t + S_0 = \underline{-16t^2 + V_0 t + S_0}$$

odd place for this consequence of FTC

FTC $$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$.

So $$\int_a^b f'(x)\,dx = f(b) - f(a)$$

i.e. the definite integral of an instantaneous rate of chg is the ACCUMULATED TOTAL CHANGE

6.7/1-14

watch out for displacement vs distance

Jan 9-8:30 AM